Example: Interpretation of $(\mathbb{Q}, +, \cdot, 0, 1)$

in $(\mathbb{Z}, +, \cdot, 0, 1)$

two-dimensional

- $\partial(x_1, x_2) := (x_2 \neq 0)$

$$h : \partial^{\mathbb{Z}} \longrightarrow \mathbb{Q}$$
$$(a,b) \longmapsto \frac{a}{b}$$

- $\varepsilon(x_1, x_2, y_1, y_2) := (x_1 y_2 = x_2 y_1)$

- $\varphi_+(x_1, x_2, y_1, y_2, z_1, z_2) := \big(z_2 (x_1 y_2 + x_2 y_1) = z_1 \cdot x_2 \cdot y_2\big)$

$$\frac{x_1}{x_2} + \frac{y_1}{y_2} = \frac{z_1}{z_2}$$

- $\varphi_\cdot(x_1, x_2, y_1, y_2, z_1, z_2) := (z_2 x_1 y_1 = z_1 x_2 y_2)$

$$\frac{x_1}{x_2} \cdot \frac{y_1}{y_2} = \frac{z_1}{z_2}$$

- $\varphi_0(x_1, x_2) := (x_1 = 0)$
- $\varphi_1(x_1, x_2) := (x_1 = x_2)$