Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
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SS 2008

Algorithmic Model Theory — Assignment 3

Due: Tuesday, 6 May, **16:00**

Exercise 1

FO^2 retains the finite model property even if we allow the use of constants and relations of higher arity than two. Prove this for a vocabulary containing a ternary relation R and a constant c as the only non-unary and non-binary symbols.

Hint: Show that c and R can be eliminated by introducing new unary and binary relations.

Exercise 2

(a) Let τ be a vocabulary containing only monadic (i.e., unary) relation symbols but no function symbols. Show that $\mathrm{FO}(\tau)$ has the finite model property.

Hint: Use Ehrenfeucht-Fraïssé games.

(b) Construct infinity axioms in the classes

(i) $[\exists\forall^2, (0), (1)]_=$ and

(ii)* $[\forall\exists, (0), (1)]_=$.

Hint: For the latter class express a property of infinite binary trees in which the edge relation is given by the unary function.

Exercise 3

Specify a conservative reduction from $[\forall^2, (0,1), (1)]$ to $[\forall^2, (1), (0,1)]$, i.e., from the class of formulae with quantifier prefix $\forall^2$ over a vocabulary containing only one binary relation and a unary function symbol to the class of formulae with the same quantifier prefix but over a vocabulary containing a unary relation and a binary function symbol. Note that equality may not be used in the formulae.

Hint: A binary relation P and a unary function f can be encoded using a unary relation Q and a binary function g in the following way: $P = \{(a,b) : g(a,b) \in Q\}$, and $f(a) = g(a,a)$.