Algorithmic Model Theory — Assignment 5

Due: Tuesday, **3 June**, 12:00

Note: Due to the RWTH Sports Day, the exercise class on Wednesday, 28 May, is cancelled.

Exercise 1

Let $\mathfrak{A}$ be a relational τ-structure with at least two elements. Show that the disjoint sum $\mathfrak{A} + \mathfrak{A}$, i.e., the τ-structure over the universe $A \times \{0\} \cup A \times \{1\}$ and with relations

$$R^{\mathfrak{A}+\mathfrak{A}} := R^{\mathfrak{A}} \times \{0\} \cup R^{\mathfrak{A}} \times \{1\} \quad \text{for all } R \in \tau,$$

is interpretable in $\mathfrak{A}$.

Exercise 2

Given a collection $I := (\partial(x), \varepsilon(x,y), \varphi_0(x), \varphi_\le(x,y), \varphi_+(x,y,z))$ of FO-formulae over a signature $\tau = \{E\}$, construct a formula ψ_I such that, for any structure $\mathfrak{B}$, we have $\mathfrak{B} \models \psi_I$ if, and only if, I induces in $\mathcal{B}$ a well defined structure $I(\mathfrak{B})$ over the signature $\sigma = \{0, \le, +\}$.

Exercise 3

For each $n < \omega$, let $T^n = (\{0, \ldots, n-1\}^*, s_0, \ldots, s_{n-1}, <)$ be the complete n-ary tree with n successor functions $s_0, \ldots, s_{n-1}$ such that $s_i(x) = xi$ for each $0 \le i < n$, and the prefix relation $<$ such that $x < y$ if $y = xz$ for some $z \in \{0, \ldots, n-1\}^*$.

In the infinitely branching tree $T^\omega = (\omega^*, \prec, <)$ each node has countably many successors that are ordered by the relation $\prec$. Again, $<$ denotes the prefix relation.

(a) Show that T^n is interpretable in the complete binary tree T^2 for each $n > 2$.

(b) Show that T^ω is interpretable in the complete binary tree T^2.