Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
Prof. Dr. E. Grädel, D. Berwanger, T. Ganzow

SS 2008

Algorithmic Model Theory — Assignment 7

Due: Tuesday, 17 June, 12:00

Exercise 1

$S1S_0$ is the fragment of S1S defined as the smallest set of formulae that contains, for a countable supply of monadic second-order variables, the atomic formulae

- $X \subseteq Y$ "X is a subset of Y",
- $\mathrm{Sing}(X)$ "X is a singleton set", and
- $\mathrm{Succ}(X, Y)$ "$X = \{x\}$, $Y = \{y\}$ and $y = s(x)$"

and that is closed under

- Boolean connectives $\vee$ and $\neg$, and
- existential monadic second-order quantification.

(a) Show that each S1S-formula φ can be effectively translated into an $S1S_0$-formula that is equivalent to φ over finite words.

(b) For each atomic $S1S_0$-formula ψ with free variables $\{X_1, \dots, X_k\}$, construct an automaton $\mathcal{A}_\psi$ over $\{0,1\}^k$ such that, for all $w \in (\{0,1\}^k)^*$, $w \in L(\mathcal{A}_\psi)$ iff $\underline{w} \models \psi(X_1, \dots, X_k)$.

Exercise 2

(a) Let φ be an arbitrary S1S-sentence over the vocabulary $\{\mathrm{succ}, (X_a)_{a\in\Sigma}\}$. Define (in S1S) the ω-language

$$L = \{\alpha \in \Sigma^\omega : \alpha \text{ contains infinitely many prefixes } w \text{ such that } \underline{w} \models \varphi\}.$$

(b) Let $\psi(x, y)$ be a first-order formula about Σ-labeled trees. Formalise in MSO that the pair (x, y) is contained in the transitive closure of the relation defined by ψ.

Exercise 3

A Σ-labeled binary tree is a function $t : \{0,1\}^* \to \Sigma$. Analogously to ω-words, we can identify a tree t with the structure $\underline{t} := (\{0,1\}^*, s_0, s_1, (X_a)_{a\in\Sigma})$ where s_0 and s_1 denote the usual successor functions, and $X_a = \{w : t(w) = a\}$. A tree-language over Σ is a set of Σ-labeled trees.

(a) Define the prefix relation in S2S.

(b) Define the following tree-languages over $\Sigma := \{a, b\}$ by S2S-formulae.

$$T_1 := \{t : \text{there is a path in } t \text{ containing infinitely many } b\},$$
$$T_2 := \{t : \text{all paths through } t \text{ contain infinitely many } b\}.$$

(c) Construct parity automata recognising T_1 and T_2 by hand, i.e. do not transform the formulae found in (b).