Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
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SS 2008

Algorithmic Model Theory — Assignment 11

Due: Tuesday, 8 July, 12:00

Exercise 1

Prove that any automatic structure $\mathfrak{A}$ admits an injective presentation, i.e., a presentation $\mathfrak{d}$ such that $\nu : L_\delta \to A$ is injective.

Exercise 2

Recall the encoding of ordered structures presented in the lecture. Let $\tau = \{P, R\}$ be a signature consisting of a unary predicate P and a binary predicate R. Construct formulae $\beta_0(\bar{x})$ and $\beta_1(\bar{x})$ defining the $\bar{x}$-th symbol of the encoding of an ordered τ-structure.

Exercise 3

(a) Show that the following classes are in NP by constructing Σ^1_1-formulae defining them.

  (i) The class of bipartite graphs,

  (ii) the class of Hamiltonian graphs, and

  (iii) the class of graphs that admit a perfect matching.

(b) Prove using Fagin's Theorem that SAT, the satisfiability problem for propositional logic, is complete for NP.

Exercise 4

The *spectrum* of a sentence $\varphi \in \mathrm{FO}(\tau)$ is defined as

$$\mathrm{spec}(\varphi) := \{n \in \mathbb{N} : \text{there exists } \mathfrak{A} \models \varphi \text{ with } |A| = n\}.$$

Show that a set $S \subseteq \mathbb{N}$ is a spectrum of an FO-formula if and only if $S \in \textsc{Nexptime}$.
Hint: Use Fagin's Theorem.