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SS 2016

Algorithmic Model Theory — Assignment 5

Due: Friday, 27 May, 13:00

Exercise 1

We say that an $\mathrm{FO}(\tau \cup \{<\})$-sentence φ is *order-invariant* if for all *finite* τ-structures $\mathfrak{A}$ and linear orderings $<, <'$ on A we have

$$(\mathfrak{A}, <) \models \varphi \Leftrightarrow (\mathfrak{A}, <') \models \varphi.$$

Show that the problem whether a given $\mathrm{FO}(\tau \cup \{<\})$-sentence φ is order-invariant is undecidable.

Hint: Show that Fin-Sat(FO) is reducible to this problem.

Exercise 2

Let τ be a fixed (finite) vocabulary which only consists of monadic relation symbols and let X be the set of all $\mathrm{FO}(\tau)$-sentences in prenex normal form.

(a) Show that $\mathrm{Sat}(X) \in \mathrm{PSPACE}$.

(b) Show that $\mathrm{Sat}(X)$ is PSPACE-complete.

Hint: Reduce QBF (the quantified Boolean formula problem) to $\mathrm{Sat}(X)$.

Exercise 3

Recall the encoding of ordered structures presented in the lecture. Let $\tau = \{P, R\}$ be a signature consisting of a unary predicate P and a binary predicate R. Construct formulae $\beta_0(\bar{x})$ and $\beta_1(\bar{x})$ defining the $\bar{x}$-th symbol of the encoding of an ordered τ-structure.

Exercise 4

(a) Show that the following classes of (undirected, finite) graphs are in NP by explicitly constructing Σ^1_1-sentences defining them.

  (i) The class of regular graphs (i.e. every node has the same number of neighbours),

  (ii) the class of Hamiltonian graphs, and

  (iii) the class of graphs that admit a perfect matching.

(b) Let $k \geq 1$. An (undirected, finite) graph $G = (V, E)$ has connectivity k if $|G| > k$ and

- for all $S \subseteq V, |S| < k$ the graph $G \setminus S$ is connected, and
- there exists a set $S \subseteq V, |S| = k$ such that $G \setminus S$ is not connected.

Construct a Σ^1_1-sentence defining the class of (undirected) graphs with connectivity k.