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SS 2016

Algorithmic Model Theory — Assignment 6

Due: Friday, 3 June, 13:00

Exercise 1

Construct an SO-HORN-sentence ψ which defines the class of (undirected) graphs $G = (V, E, c, d)$ (with two constant symbols c and d) in which there is no path from c to d.

Exercise 2

To justify the definition of SO-HORN, show that the admission of arbitrary first-order prefixes would make the restriction to Horn clauses pointless. Show that this extension of SO-HORN has the full power of second-order logic.

Exercise 3

weak-SO-HORN is the subclasss of SO-HORN consisting of all sentences of the form

$$QR_1 \dots QR_k \forall x_1, \dots \forall x_l \bigwedge_{1 \le i \le r} C_i,$$

where the clauses C_i are of the form $\beta_1 \wedge \dots \wedge \beta_n \to H$ and where the β_i *are either atoms or negated atoms* with the restriction that the relations $R_1, \dots, R_k$ only occur positively. In other words, weak-SO-HORN differs from SO-HORN in the fact that only atomic or negated atomic first-order formulas are allowed in the clauses (instead of arbitrary first-order formulas which do not contain $R_1, \dots, R_k$).

(a) Show that on ordered structures weak-SO-HORN is strictly less expressive than SO-HORN.

Hint: Show that for every weak-SO-HORN sentence ψ the class $\{\mathfrak{A} : \mathfrak{A} \models \psi\}$ is closed under substructures.

(b) Show that, however, on ordered structures with the additional successor relation and constants $0, e$ for the first and last element in the order weak-SO-HORN and SO-HORN are equally expressive.

Hint: Show that on this domain weak-SO-HORN captures PTIME.