Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
Prof. Dr. E. Grädel, W. Pakusa, F. Reinhardt,
M. Voit

SS 2016

Algorithmic Model Theory — Assignment 9

Due: Friday, 24 June, 13:00

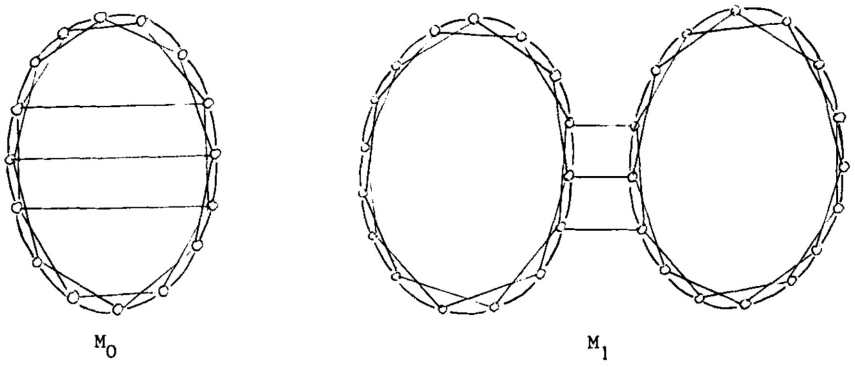
Exercise 1

An (undirected) graph $G = (V, E)$ is k-edge-connected if the removal of any set of at most $k-1$ edges does not disconnect the graph. Show that for all $k \geq 2$ there is no sentence $\psi_k \in \mathrm{FO}(E)$ such that for all $(k-1)$-edge-connected graphs G:

$$G \models \psi_k \quad \Leftrightarrow \quad G \text{ is } k\text{-edge-connected.}$$

(That is FO cannot axiomatise k-edge-connectivity inside the class of $(k-1)$-edge-connected graphs).

Hint:



Exercise 2

Determine the asymptotic probabilities of the following graph properties.

(i) $\mathcal{K}_1 = \{G : G \text{ has no isolated node}\}$

(ii) $\mathcal{K}_2 = \{G : G \text{ is bipartite}\}$

(iii) $\mathcal{K}_3 = \{G : G \text{ is a tree}\}$

(iv) $\mathcal{K}_4 = \{G : G = (V, E) \text{ contains a clique of size } \geq \log(|V|)\}$

(v) $\mathcal{K}_5 = \{G : G \text{ contains an even number of edges}\}$

Exercise 3

Prove or disprove that the following logics have the zero-one law with respect to the uniform probability distribution on the respective classes. ($[n] := \{0, 1, \ldots, n-1\}$)

(i) FO over the class of finite linear orders $\mathrm{Lin} = \{([n], <) : n \in \mathbb{N}, < \text{ linear order on } [n]\}$.

(ii) MSO over the class of finite linear orders Lin.

(iii) FO over the class of finite binary words $\mathrm{W} = \{([n], <, P) : ([n], <) \in \mathrm{Lin}, P \subseteq [n]\}$.

(iv) SO over the class of all graphs.