Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
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SS 2011

Algorithmic Model Theory — Assignment 1

Due: Monday, 24 October, 12:00

Note:
- You may work on the exercises in groups of up to three students.
- Hand in your solutions at the end of the lecture or put them into the box at the institute.

Exercise 1

(a) Show that any two disjoint co-recursively enumerable languages A and B are recursively separable, i.e. there exists a recursive set C such that $A \subseteq C$ and $B \cap C = \emptyset$.

(b) Given a recursively enumerable language L, let $\operatorname{code} L = \{\rho(M) \ : \ L(M) = L\}$. Show that if L_1 and L_2 are recursively enumerable languages and $L_1 \subsetneq L_2$, then $\operatorname{code} L_1$ is recursively inseparable from $\operatorname{code} L_2$.

Hint: Use a reduction from a suitable pair of recursively inseparable sets and recall the proof of Rice's theorem.

Exercise 2

Let X be the set of relational FO-sentences of the form $\exists x_1 \dots \exists x_r \forall y_1 \dots \forall y_s \, \varphi$ where $r, s \in \mathbb{N}$ and φ is quantifier-free. Show that $\mathrm{Sat}(X)$ is decidable.
Hint: Show that each satisfiable sentence in X has a model with at most r elements.