Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
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Algorithmic Model Theory — Assignment 5

Due: Monday, 21 November, 12:00

Exercise 1

Show that the class $[\exists^*\forall, (0), (1)]_=$ has the finite model property.

Hint: Consider the Skolem normal-form of such sentences φ, and try to prune a possibly infinite model of φ by considering equivalence relations between elements of the structure relating those elements that satisfy the same atomic formulae in one free variable in which the function is applied only a bounded number of times.

Exercise 2

(a) Show, using the arguments from Exercise 2 of Assignment 1, that $\text{Sat}([\exists^*\forall^*, all, (0)]_=) \in$ NEXPTIME.

(b) Show that $\text{Sat}([\exists^*\forall^*, all, (0)]_=)$ is NEXPTIME-complete by proving the hardness via a reduction from $\text{DOMINO}(\mathcal{D}, 2^n)$ to $\text{Sat}([\exists^2\forall^*, all, (0)]_=)$.

Hint: Use sentences of the form $\exists 0 \exists 1 \forall \bar{x} \forall \bar{y} \dots (0 \neq 1 \land \varphi)$ where tuples $\bar{x} = x_0 \dots x_{n-1}$ represent coordinates and φ describes a correct tiling using appropriate relations.

Exercise 3

(a) Show that $\text{Sat}([all, (m), (0)]_=) \in$ PSPACE for every fixed $m \in \mathbb{N}$.

(b) Show that $\text{Sat}([all, (m), (0)]_=)$ is PSPACE-complete.
Hint: Reduce QBF (the set of all valid quantified boolean formulae) to $\text{Sat}([all, (0), (0)]_=)$ i.e. the first-order theory of equality.