Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
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Algorithmic Model Theory — Assignment 10

Due: Monday, 9 January, 12:00

Exercise 1

Conway's LIFE is a game played on an undirected graph. At the beginning some vertices are marked with a pebble. In every turn the following rules are applied simultaneously to all vertices:

- marked vertices remain marked if and only if they have 2 or 3 neighbours;
- unmarked vertices become marked if and only if they have exactly 3 neighbours.

Write a PFP formula over the signature $\{E, P\}$ (E is the edge relation and P the set of vertices marked at the beginning) which holds in an arena $\mathcal{G} = (V, E^{\mathcal{G}}, P^{\mathcal{G}})$ if and only if the game becomes eventually stationary.

Exercise 2

Let $\mathcal{K}$ be a class of τ-structures with the following property. For every $m \in \mathbb{N}$, there exists a structure $\mathfrak{A} \in \mathcal{K}$ such that for all m-tuples $\overline{a}$ in $\mathfrak{A}$ there exists a non-trivial automorphism of $(\mathfrak{A}, \overline{a})$. Show that $\mathcal{K}$ does not admit definable orders (even with parameters) in any logic considered in this lecture.

Exercise 3

Let $\mathcal{D}$ be the domain of structures $(A, E, R_1, \ldots)$ such that (A, E) is isomorphic to a finite rectangular undirected grid. Show that LFP captures PTIME on $\mathcal{D}$.

Exercise 4

Construct formulae of the multidimensional μ-calculus that define the following classes C of rooted transition systems:

- $C = \{(\mathcal{G}, v) :$ from v a terminal vertex is reachable that satisfies the same atomic propositions$\}$
- $C = \{(\mathcal{G}, v) :$ there are two infinite paths π and σ starting from v such that for all positions $i > 0$ and all predicates P it holds $(\mathcal{G}, \pi[i]) \models P$ iff $(\mathcal{G}, \sigma[i]) \not\models P\}$