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Mathematische Grundlagen der Informatik
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Algorithmic Model Theory — Assignment 7

Due: Monday, 9 December, 12:00

Exercise 1

Construct an LFP-formula φ such that a (finite) undirected graph $\mathcal{G}$ is a model of φ if, and only if, $\mathcal{G}$ is bipartite.

Hint: A graph is bipartite if, and only if, it does not contain a cycle of odd length.

Exercise 2

Let $\mathcal{G} = (V, E, P)$ be a finite directed graph extended by a monadic predicate P. Describe the relations in $\mathcal{G}$ that are defined by the following LFP-formulae.

(a) $[\mathbf{gfp}\, Rx.\, (Px \wedge \exists y(Exy \wedge [\mathbf{lfp}\, Sx.\, (Rx \vee \exists y(Exy \wedge Sy))]\,(y))\,]\,(x)$

(b) $[\mathbf{lfp}\, Uxy.\, (Exy \vee \exists z(Exz \wedge [\mathbf{lfp}\, Gxy.\, (\exists z(Exz \wedge Uzy))]\,(z, y))\,]\,(x, y)$

Exercise 3

For $n \geq 1$ we consider the directed $(n \times n)$-grid as a relational structure $\mathcal{G}_n = (U_n, H_n, V_n)$ over the signature $\tau = \{H, V\}$ where

- $U_n = \{(i, j) : 0 \leq i, j \leq n - 1\}$, and
- $H_n = \{((i, j), (i + 1, j)) : 0 \leq j \leq n - 1,\ 0 \leq i < n - 1\}$, and
- $V_n = \{((i, j), (i, j + 1)) : 0 \leq i \leq n - 1,\ 0 \leq j < n - 1\}$.

Construct an LFP-formula $\varphi(x, y)$ which defines a linear order on the class of all $(n \times n)$-grids, i.e. $\varphi^{\mathcal{G}_n} = \{(a, b) : \mathcal{G}_n \models \varphi(a, b)\}$ is a linear order on U_n for all $n \geq 1$.

Exercise 4

(a) Construct an LFP-formula $\varphi(p, x, y)$ such that for all directed cycles $\mathcal{C} = (V, E)$ and every $v \in V$ the relation $\varphi(v)^{\mathcal{C}} = \{(a, b) : \mathcal{C} \models \varphi(v, a, b)\}$ is a linear order on V.

(b) Prove that the parameter p is necessary, i.e. prove that there is no LFP-formula $\varphi(x, y)$ which defines a linear order on the class of all directed cycles.

Exercise 5

Prove that on the classes of structures from Exercise 3 and 4 (i.e. on the class of directed grids and on the class of directed cycles) LFP captures PTIME.