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Algorithmic Model Theory — Assignment 10

Due: Monday, 13 January, 12:00

Exercise 1

Show that FPC $\leq \mathrm{C}^{\omega}_{\infty\omega}$ (for formulas without free numerical variables).
Hints:

- Construct for every formula $\varphi(\bar{x}, \bar{\nu}) \in$ FPC and $n \in \omega, \bar{\nu} \in \{0, \ldots, n-1\}^k$ a formula $\varphi^{\star}_{n,\bar{\nu}}(\bar{x})$ which is equivalent to φ on structures of size n, i.e. for all $\mathfrak{A}$ of size n we have

$$\mathfrak{A} \models \varphi(\bar{a}, \bar{\nu}) \text{ iff } \mathfrak{A} \models \varphi^{\star}_{n,\bar{\nu}}(\bar{a}), \quad \text{for all } \bar{a} \in A.$$

- For fixed point operators, adapt the construction from the proof showing FP $\leq \mathrm{L}^{\omega}_{\infty\omega}$.

Exercise 2

In the lecture, the k-pebble bijection game k-BG$(\mathfrak{A}, \mathfrak{B})$ was introduced which characterises $C^k_{\infty\omega}$-equivalence of structures.

(a) Modify the rules of the game to capture equivalence in $L^k_{\infty\omega}$ rather than $C^k_{\infty\omega}$.
Hint: Relax the requirement for Duplicator to choose a bijection.

(b) Use this game to show that the following classes of structures are undefinable in FP:
- The class of (undirected) graphs with an Eulerian cycle.
  Hint: Consider complete graphs.
- The class of (undirected) graphs with an Hamiltonian cycle.
  Hint: Consider complete bipartite graphs.

Exercise 3

Let $G = (V, E)$ be a finite undirected graph and $v, w \in V$.
Show that Duplicator has a winning strategy in the 2-pebble bijection game 2-BG(G,G) starting from position (v, w) if v and w have the same colour in the stable colouring of G (cf. assignment 9, exercise 4).