Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
Prof. Dr. E. Grädel, K. Dannert

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Algorithmic Model Theory — Assignment 4

Due: Thursday, 7 November, 10:30

Exercise 1 — 5 Points

We say that an $\mathrm{FO}(\tau \cup \{<\})$-sentence φ is *order-invariant* if for all *finite* τ-structures $\mathfrak{A}$ and linear orderings $<, <'$ on A we have

$$(\mathfrak{A}, <) \models \varphi \quad \Leftrightarrow \quad (\mathfrak{A}, <') \models \varphi.$$

Show that the problem whether a given $\mathrm{FO}(\tau \cup \{<\})$-sentence φ is order-invariant is undecidable.

Hint: Show that Fin-Sat(FO) is reducible to this problem.

Exercise 2 — 10 Points

Let τ be a fixed (finite) vocabluary which only consists of monadic relation symbols and let X be the set of all $\mathrm{FO}(\tau)$-sentences in prenex normal form.

(i) Show that $\mathrm{Sat}(X)$ is in PSPACE.

(ii) Show that $\mathrm{Sat}(X)$ is PSPACE-complete.

Hint: Reduce QBF (the quantified Boolean formula problem) to $\mathrm{Sat}(X)$.

Exercise 3 — 15 Points

(a) Show that the following classes of (undirected, finite) graphs are in NP by explicitly constructing Σ^1_1-sentences defining them.

  (i) The class of regular graphs (i.e. every node has the same number of neighbours).

  (ii) The class of Hamiltonian graphs.

  (iii) The class of graphs that admit a perfect matching.

(b) Let $k \geq 1$. An (undirected, finite) graph $G = (V, E)$ has *connectivity* k if $|G| > k$ and

  - for all $S \subseteq V,\ |S| < k$ the graph $G \backslash S$ is connected, and
  - there exists a set $S \subseteq V,\ |S| = k$ such that $G \backslash S$ is not connected.

  Construct a Σ^1_1-sentence defining the class of (undirected) graphs with connectivity k.

(c) Construct an SO-HORN-sentence which defines the class of (undirected) graphs $G = (V, E, c, d)$ (with constant symbols c and d) in which there is no path from c to d.