**Lehr- und Forschungsgebiet**
**Mathematische Grundlagen der Informatik**
RWTH Aachen
Prof. Dr. E. Grädel, R. Wilke

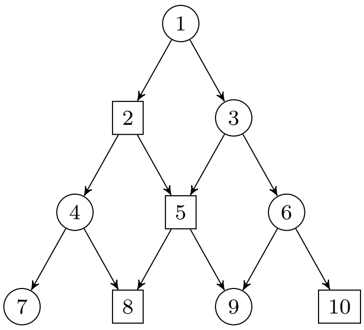
WS 2018/19

# Logic and Games — Assignment 1

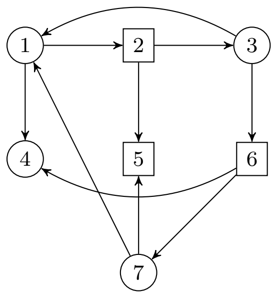
Due: Tuesday the 16th at 12:00 in the lecture or at our chair.

## Exercise 1 — 4 Points

Consider the game graphs $\mathcal{G}_i = (V^i, V_0^i, V_1^i, E^i)$, where ⓙ denotes a position of player 0 and [k] one of player 1.




$\mathcal{G}_1$




$\mathcal{G}_2$

Compute the winning regions $W_0$ and $W_1$ in both games. Infinite plays are considered as ties.

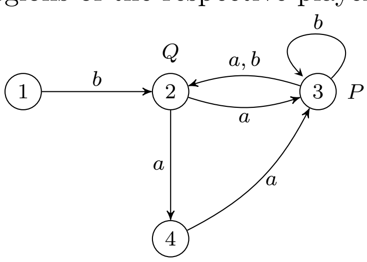
## Exercise 2 — 9 Points

Evaluate the following ML-formulae on the given Kripke-structure by constructing the model-checking game and computing the winning regions of the respective players.

(a) $\varphi_a = \langle b \rangle \langle a \rangle P$;

(b) $\varphi_b = [a]\langle a \rangle (Q \vee \langle b \rangle P)$;

(c) $\varphi_c = [b]\,([b]P \wedge \langle a \rangle \neg P)$.




## Exercise 3 — 8 Points

Let $\mathcal{G} = (V, V_0, V_1, E)$ be a reachability game. Consider the two inductive definitions of the winning regions:

(i)
$$W_\sigma^0 := \{v \in V_{1-\sigma} \;:\; vE = \varnothing\}$$
$$W_\sigma^{n+1} := \{v \in V_\sigma \;:\; vE \cap W_\sigma^n \neq \varnothing\} \cup \{v \in V_{1-\sigma} \;:\; vE \subseteq W_\sigma^n\}$$

(ii)
$$\widetilde{W}_\sigma^0 := \{v \in V_{1-\sigma} \;:\; vE = \varnothing\}$$
$$\widetilde{W}_\sigma^{n+1} := \widetilde{W}_\sigma^n \cup \left\{v \in V_\sigma \;:\; vE \cap \widetilde{W}_\sigma^n \neq \varnothing\right\} \cup \left\{v \in V_{1-\sigma} \;:\; vE \subseteq \widetilde{W}_\sigma^n\right\}$$

(a) Show that $W_\sigma^n = \widetilde{W}_\sigma^n$ holds for all $n \in \mathbb{N}$.

(b) If $\mathcal{G}$ is a *finite* reachability game (that means, $|V| \in \mathbb{N}$), then

$$W_\sigma = \bigcup_{n \in \mathbb{N}} W_\sigma^n.$$

In *infinite* reachability games only "$\supseteq$" holds. Provide a counterexample showing that inequality is possible!

**Exercise 4** 9 Points

The $(n,k)$-*Nim-game* is played with $n$ matches, given ($n \geq k \geq 1$). Two players alternate their turns, where the current one removes at least 1 and at most $k$ matches from the board. If the last match is removed, the current player loses. Otherwise it is the opponents turn.

For which choices of $n$ and $k$ does the starting player win the $(n,k)$-Nim-game?