Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
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Logic and Games — Assignment 5

Due: Tuesday the 20th November at 12:00 in the lecture or at our chair.

Exercise 1 4 Points

An operator $F : \mathcal{P}(A) \to \mathcal{P}(A)$ is called *inflationary*, if $F(X) \supseteq X$ for all $X \subseteq A$. Give examples for operators $F : \mathcal{P}(A) \to \mathcal{P}(A)$ with the following properties:

(a) F has a fixed point but no least fixed point.

(b) F has a least fixed point but F is not monotone.

(c) F is monotone but not inflationary.

(d) F is inflationary but not monotone.

Exercise 2 12 Points

Consider the signature $\tau = \{P, Q\}$. Give an L_μ-formula $\varphi \in \mathrm{L}_\mu(\tau)$ such that for each transition system $\mathcal{K} = (V, E, P, Q)$ and each node $v \in V$ we have $\mathcal{K}, v \models \varphi$ if and only if

(a) at each node reachable from v where Q holds, P holds as well.

(b) from each node reachable from v where P holds, there is a reachable node where Q holds.

(c) there is an infinite path from v such that $P \wedge Q$ holds only finitely many times.

Exercise 3 8 Points

Let $\mathfrak{N} = (\mathbb{N}, S, 0)$ and $\mathfrak{Z} = (\mathbb{Z}, S, 0)$, where S denotes the successor function on $\mathbb{N}$ and $\mathbb{Z}$, respectively.

(a) Define the relations $+ \subseteq \mathbb{N}^3$ and $\cdot \subseteq \mathbb{N}^3$ in LFP.

(b) Define the relation $< \subseteq \mathbb{Z}^2$ in LFP.

Exercise 4 6 Points

Prove that the following problem is undecidable:

- Let $\varphi(x)$ be a formula in FO.
- Is $F_\varphi^\mathfrak{A}$ monotone for all structures $\mathfrak{A}$ with signature $\tau(\varphi)\setminus\{R\}$?

Hint: Use the fact that the satisfiability problem for FO is undecidable.