Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
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WS 2010/11

Mathematical Logic II — Assignment 13

Due: Monday, January 31, 12:00

Exercise 1 3 Points

The *weak monadic second-order logic* wMSO is an extension of FO by quantifiers over *finite* sets. Prove or disprove that for every wMSO-sentence φ there is an $L_{\omega_1\omega}$-sentence ψ that has the same models as φ.

Exercise 2 5 Points

Let $\mathfrak{A}$ and $\mathfrak{B}$ be ω-homogeneous τ-structures. Prove that $\mathfrak{A} \equiv_\infty \mathfrak{B}$ holds if and only if each type of $\mathfrak{A}$ over the empty set that is realised in $\mathfrak{A}$ is also a type of $\mathfrak{B}$ that is realised in $\mathfrak{B}$, and vice versa.

Exercise 3 3 + 2 + 5* Points

Consider the signature $\tau = \{E, P, R\}$ with relational symbols E and R of arity 2 and P of arity 1.

(a) Describe the class of graphs $\mathcal{G}$ that satisfy the formula

$$\varphi(x, y) := \forall x'(Exx' \to \exists y'(Eyy' \land Rx'y')) \land \forall y'(Eyy' \to \exists x'(Exx' \land Rx'y'))$$

(b) Write an LFP-formula $\varphi(x) \in \mathrm{LFP}(\tau)$ such that for each directed graph $\mathcal{G} = (V, E^{\mathcal{G}}, P^{\mathcal{G}})$ and each vertex $v \in V$, $\mathcal{G} \models \varphi(v)$ holds if and only if every terminal vertex that is reachable from v in $\mathcal{G}$ is contained in $P^{\mathcal{G}}$.

(c*) Write an LFP-formula $\varphi(x) \in \mathrm{LFP}(\tau)$ such that for each directed graph $\mathcal{G} = (V, E^{\mathcal{G}}, P^{\mathcal{G}})$ and each vertex $v \in V$, $\mathcal{G} \models \varphi(v)$ holds if and only if there is an infinite path from v in $\mathcal{G}$ with only finitely many vertices in $P^{\mathcal{G}}$.