Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
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SS 2015

Quantum Computing — Assignment 1

Due: Wednesday, 22.04., 14:15

Exercise 1 5 Points

Consider the polarisation experiment from the lecture, where, instead of 45°, the filter in the middle polarises light with an angle α, for arbitrary α.

Specify how much light passes through the three filters depending on the angle α.

Exercise 2 15 Points

(a) Give a construction of an equation system $\mathrm{EQ}\,|\psi\rangle$ for each vector $|\psi\rangle \in H_4$, such that $\mathrm{EQ}\,|\psi\rangle$ is solvable if, and only if $|\psi\rangle$ is not entangled.

(b) For each of the following states, prove or disprove that it is entangled.

Hint: The equation system may not be helpful in all cases. You can, for instance, consider the properties of tensor products.

(i) $\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{\sqrt{2}}|11\rangle$

(ii) $\left(-\frac{1}{6} + i\frac{\sqrt{2}}{3}\right)|00\rangle + \left(\frac{1-\sqrt{2}}{6} + i\frac{1+\sqrt{2}}{6}\right)|01\rangle + \left(\frac{1-\sqrt{2}}{6} + i\frac{1+\sqrt{2}}{6}\right)|10\rangle + \left(i\frac{1}{3}\right)|11\rangle$

(iii) $\frac{3}{8}|00\rangle + \frac{\sqrt{7}}{8}|01\rangle + \frac{3}{4}|10\rangle + \frac{\sqrt{3}}{4}|11\rangle$

Exercise 3 10 Points

Show that, for each qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, there are $\gamma, \vartheta, \varphi \in \mathbb{R}$ such that

$$|\psi\rangle = e^{i\gamma}\left(\cos\frac{\vartheta}{2}|0\rangle + e^{-i\varphi}\sin\frac{\vartheta}{2}|1\rangle\right).$$

Hint: Each complex number z can be written as $z = r \cdot e^{i\gamma}$ for some $r, \gamma \in \mathbb{R}$.