Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
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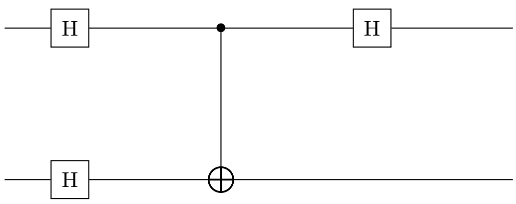
SS 2015

Quantum Computing — Assignment 3

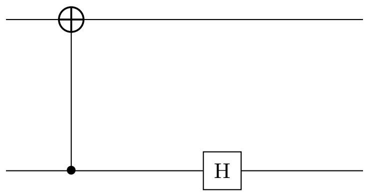
Due: Wednesday, 06.05., 14:15

Exercise 1 10 Points

(a) Prove that the following gates are equivalent:



and



(b) The Fredkin gate realizes the unitary transformation $F : H^3 \mapsto H^3$, where

$$F(|xyz\rangle) = \begin{cases} |xzy\rangle & \text{, if } x = |1\rangle \\ |xyz\rangle & \text{, otherwise} \end{cases}$$

for the base vectors.

Give a representation of the Fredkin gate using only CNOT, C-V and C-V^*, where V is the gate corresponding to $\sqrt{M_\neg}$.

Exercise 2 10 Points

Suppose Alice and Bob plan to travel to different countries. They want to prepare for the incident that, during their journeys, they want to simulate a coin flip. However, Alice and Bob do not trust each other to honestly report the outcome of a coin flip. Fortunately, Alice and Bob can take with them one qubit each, belonging to the same two-qubit state.

Give a quantum gate that, for each base vector, constructs a two-qubit state such that, when Alice measures her qubit, it is $|0\rangle$ with probability $\frac{1}{2}$, but when Bob measures his qubit afterwards, the measurement is completely determined by Alice's.

Exercise 3 10 Points

Design a quantum gate array realizing the unitary transformation

$$U_k : |0^k\rangle \mapsto \frac{1}{\sqrt{k}} \left(|0^k\rangle + \sum_{i=1}^{k-1} |0^{i-1}10^{k-i}\rangle \right).$$

Use gates corresponding to the following unitary matrices:

$$A_k = \frac{1}{\sqrt{k+1}} \begin{pmatrix} 1 & -\sqrt{k} \\ \sqrt{k} & 1 \end{pmatrix}$$

and

$$T_{j,k} = \frac{1}{\sqrt{k-j+1}} \begin{pmatrix} \sqrt{k-j+1} & 0 & 0 & 0 \\ 0 & 1 & \sqrt{k-j} & 0 \\ 0 & -\sqrt{k-j} & 1 & 0 \\ 0 & 0 & 0 & \sqrt{k-j+1} \end{pmatrix}.$$

Hint: Consider how the gates modify the probabilities.