Lehr- und Forschungsgebiet
Mathematische Grundlagen der Informatik
RWTH Aachen
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SS 2015

Quantum Computing — Assignment 7

Due: Wednesday, 17.06., 14:15

Geben Sie bitte Namen, Matrikelnummer und die Übungsgruppe an.

Exercise 1 10 Points

Determine the group of characters of $(\mathbb{Z}, +)$. Is this group isomorphic to $(\mathbb{Z}, +)$?

Exercise 2 10 Points

(a) Let $G = \{g_1, \dots, g_n\}$ be an abelian group and let $i \in \{1, \dots, n\}$. Find the Fourier transformed of $f : G \to \mathbb{C}$ defined by

$$f(g) = \begin{cases} 1, & \text{if } g = g_i \\ 0, & \text{otherwise.} \end{cases}$$

(b) We consider the operator $S : \mathbb{C}^{\mathbb{Z}_n} \to \mathbb{C}^{\mathbb{Z}_n}$ given by $S(f)(i) = f(i + 1 \mod n)$. Describe the Fourier transformed $\hat{S}(f)$ in terms of $\hat{f}$.

Exercise 3 10 Points

In the following let $n = n_0 n_1$, where $\gcd(n_0, n_1) = 1$.

(a) Let $f : \mathbb{Z}_{n_0} \times \mathbb{Z}_{n_1} \to \mathbb{Z}_n$ be the function given by $f(k_0, k_1) = a_1 n_1 k_0 + a_0 n_0 k_1$, where a_i is the multiplicative inverse of n_i modulo n_{1-i}. Show that f is an isomorphism.

(b) Prove that the Hilbert-Space $H_{\mathbb{Z}_n}$ is isomorphic to $H_{\mathbb{Z}_{n_1}} \otimes H_{\mathbb{Z}_{n_2}}$.