Winning Games with Semiring Provenance

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Lightning Math Night, Aachen 2024

How to win chocolate?

Hangry Hanna

Selfish Simon

How to win chocolate?

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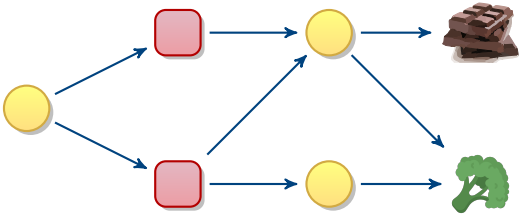
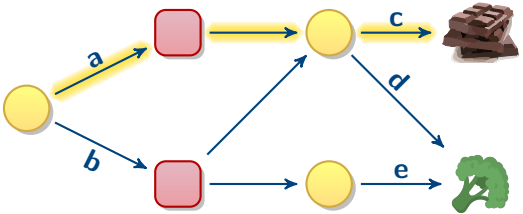
Hangry Hanna

Selfish Simon

How to win chocolate?



How to win chocolate?



Hanna's strategy: moves **a** and **c**.

How to win **more** chocolate?

Hangry Hanna
M.Sc. RWTH

Selfish Simon
M.Sc. RWTH

How to win **more** chocolate?

Hangry Hanna
M.Sc. RWTH

Selfish Simon
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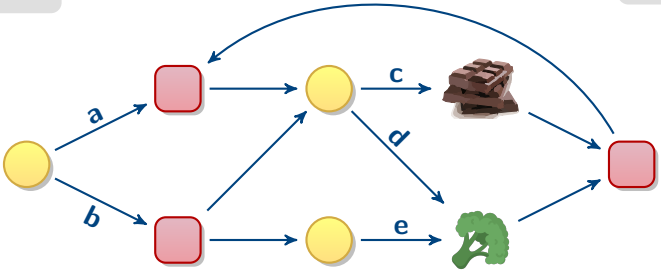
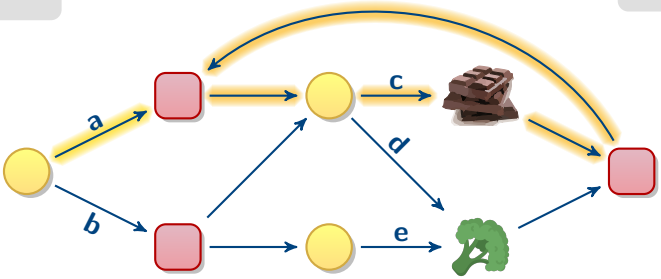
How to win **more** chocolate?

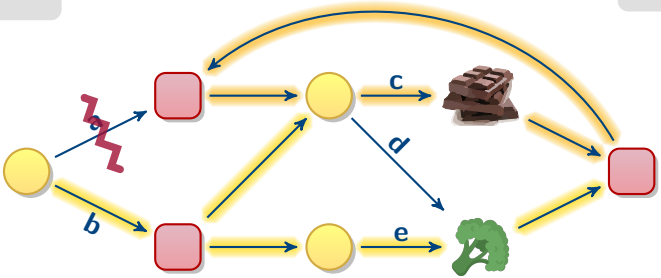


How to win **more** chocolate?



Strategy 1: **a** once
c infinitely often

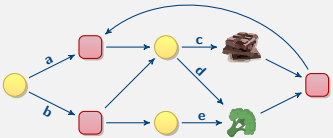
How to win **more** chocolate?



Strategy 1: **a** once
c infinitely often

Strategy 2: **b**, **e** once (at most)
c infinitely often

Unified approach to winning chocolate



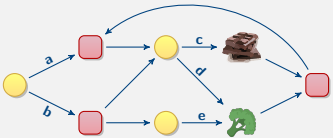
Once:

$$\mu X.\ \blacksquare \vee (\bigcirc \wedge \Diamond X) \vee (\square \wedge \Box X)$$

Infinitely often:

$$\nu Y.\ \mu Z.\ \Big(\blacksquare \wedge ((\bigcirc \wedge \Diamond Y) \vee (\square \wedge \Box Y))\Big) \vee \Big(\neg\blacksquare \wedge ((\bigcirc \wedge \Diamond Z) \vee (\square \wedge \Box Z))\Big)$$

Unified approach to winning chocolate



Once:

$\mu X.\ \blacksquare \vee (\bigcirc \wedge \Diamond X) \vee (\square \wedge \Box X) \quad \rightsquigarrow \quad a \cdot c$

Infinitely often:

$\nu Y.\ \mu Z.\ \left(\blacksquare \wedge ((\bigcirc \wedge \Diamond Y) \vee (\square \wedge \Box Y))\right) \vee \left(\neg\blacksquare \wedge ((\bigcirc \wedge \Diamond Z) \vee (\square \wedge \Box Z))\right) \quad \rightsquigarrow \quad a \cdot c^{\infty} + b \cdot e \cdot c^{\infty}$

Semiring Provenance for Fixed-Point Logics

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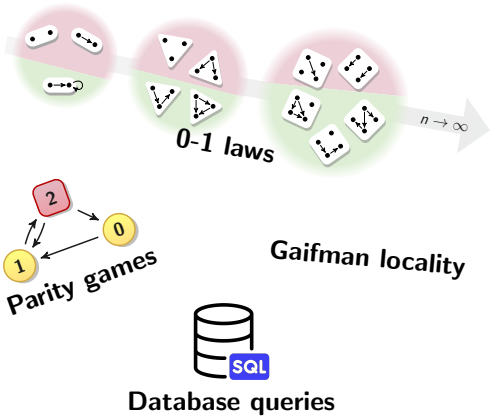
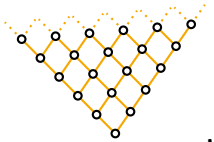


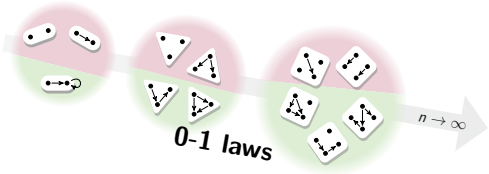
Semiring Provenance for Fixed-Point Logics

$a + ab = a$

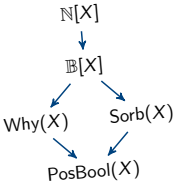
Absorption



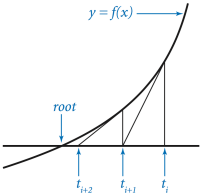
Continuous posets



0-1 laws



Polynomial semirings



Newton's method

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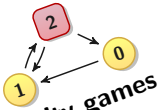


Parity games

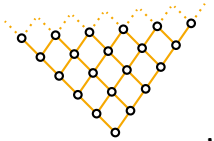
Gaifman locality

Database queries

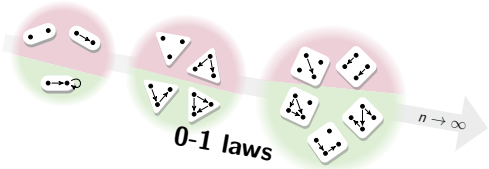
Semiring Provenance for Fixed-Point Logics

$a + ab = a$

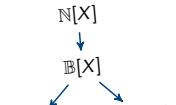
Absorption

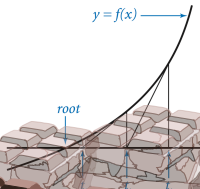


Continuous posets



0-1 laws





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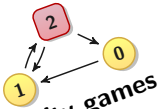

Parity games

Gaifman locality